Charged Rotating Black Holes in 5d Einstein-Maxwell-(A)dS Gravity

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Abstract

We obtain charged rotating black hole solutions to the theory of Einstein-Maxwell gravity with cosmological constant in five dimensions. Some of the physical properties of these black holes are discussed.

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1 Introduction

There has been some renewed interest in the study of black hole physics in gravitational theories with cosmological constant. Theories with negative cosmological constant can be embedded in a supersymmetric setting obtaining gauged supergravity theories in various dimensions. Gauged supergravities admit anti-de Sitter space as a vacuum state, and thus black hole solutions of these theories are of physical relevance to the proposed AdS/CFT correspondence [1]. In particular, the study of AdS black holes can give new insights into the nonperturbative structure of some conformal field theories.

On the other hand, black holes in backgrounds with positive cosmological constant, i. e. in de Sitter spaces, have also attracted some interest recently, due to the phenomenon of black hole anti-evaporation [2]. Besides, these black holes could be relevant to the proposed duality between the large N limit of Euclidean four-dimensional $U(N)$ super-Yang-Mills theory and the so-called type IIB* string theory in de Sitter space [3].

In five dimensions, extremal solutions of Einstein-Maxwell gravity were discussed in [4][1]. These solution are supersymmetric in the sense that they preserve half of the supersymmetry when viewed as bosonic solutions of the pure $N = 2$ ungauged supergravity theory. The metric for these black holes is of the Tangherlini form [7]. Later, extreme black holes for $N = 2$ ungauged supergravity coupled to abelian vector multiplets were considered in [8, 9, 10]. An important feature of these black holes is that, for those with non-singular horizons, the entropy can be expressed in terms of the extremum of the central charge. Black hole solutions of gauged supergravity theory have also been the subject of many recent studies. It is known from the results of [11] that supersymmetric solutions of this theory (with negative cosmological constant) have naked singularities, and therefore one need to study non-extremal or possibly supersymmetric rotating solutions[2]. The non-extremal generalizations of the solutions considered in [11] were studied in [14]. General rotating charged solutions in five-dimensional (anti)de Sitter spaces are not known yet. It is our purpose in this work to study these solutions, and as a first step in this direction, we will be mainly concerned with the solutions of five-dimensional Einstein-Maxwell gravity with Chern-Simons term and with a positive or negative cosmological constant.

Anti-de Sitter rotating solutions without charge in five dimensions have been recently found by Hawking et al. [15]. These solutions break all supersymmetries when viewed as solutions of $N = 2$ gauged supergravity, unless they are massless. In order to have non-trivial rotating solutions preserving some supersymmetry, it is thus necessary to include gauge fields.

This work is organized as follows. In the next section, we present our rotating solution to Einstein-Maxwell gravity with cosmological constant. In section 3, we study some physical properties of the charged rotating de Sitter black holes, like horizons, Hawking temperature and entropy. Furthermore, we find a Smarr formula, from which a first law of black hole mechanics can be derived. In section 4 we express our solution in so-called

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[1]Cf. also [5, 6] for further discussions of five-dimensional rotating black holes.

[2]In four dimensions, it has indeed been shown that rotating BPS solutions can have an event horizon [12]. For further studies of supersymmetric AdS_4 black holes cf. [13].

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cosmological coordinates, which will allow us to find also multi-centered rotating charged de Sitter black holes in five dimensions. We conclude with some final remarks.

2   Rotating charged black hole solutions in (anti)de Sitter space

In this section we will present new rotating charged solutions in five dimensional (anti)-de Sitter spaces. Consider the Einstein-Maxwell theory in five dimensions with cosmological constant. This theory contains the graviton and one abelian gauge field. The action is given by [3]

$$S = \frac{1}{4\pi G_5} \int d^5 x \, e \left(\frac{1}{2} R - \Lambda - \frac{1}{24} F_{\mu\nu} F^{\mu\nu} + \frac{e^{-1}}{216} \epsilon^{\mu\nu\rho\sigma\lambda} F_{\mu\nu} F_{\rho\sigma} A_\lambda \right), \tag{2.1}$$

where μ, ν are spacetime indices, R is the scalar curvature, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ denotes the abelian field-strength tensor, and $e = \sqrt{-g}$ is the determinant of the fünfbein e^a_μ. G_5 denotes the five-dimensional Newton constant, and $\Lambda = -6g^2$ is the cosmological constant. In the context of supergravity, the gauging of the theory enforces the addition of a cosmological constant with g being the coupling constant of the gravitino to the gauge field. This means that for negative Λ, the action (2.1) can be regarded as the bosonic part of the gauged supersymmetric five-dimensional $N = 2$ supergravity theory without vector multiplets.

The Einstein and gauge field equations of motion derived from (2.1) read

$$R_{\mu\nu} = F^2_{\mu\nu} - \frac{1}{6} g_{\mu\nu} F^2 + 4g^2 g_{\mu\nu}, \tag{2.2}$$

$$\partial_\nu (e g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma}) = \frac{1}{12} \epsilon^{\mu\nu\rho\sigma\kappa} F_{\nu\rho} F_{\sigma\kappa}, \tag{2.3}$$

where $F^2 = \frac{1}{6} F_{\mu\nu} F^{\mu\nu}$ and $F^2_{\mu\nu} = \frac{1}{6} g^{\rho\lambda} F_{\mu\rho} F_{\nu\lambda}$.

The above equations admit the (anti) de Sitter vacuum as a solution which (for negative Λ) is maximally supersymmetric. This solution has zero gauge fields, and the metric can be represented by[4]

$$ds^2 = -(1 + g^2 r^2) dt^2 + \frac{1}{1 + g^2 r^2} dr^2 + r^2 d\Omega^2, \tag{2.4}$$

where $d\Omega^2$ denotes the standard metric on the unit three-sphere.

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[3]Our action is related to that of [16] by the rescaling of the gauge fields of [16] by $\frac{1}{2\sqrt{3}}$ and multiplying the action by an overall factor of 2. We use the metric $\eta_{ab} = (-, +, +, +, +)$.

[4]For g real (negative cosmological constant) we have the anti-de Sitter solution and for imaginary g (positive cosmological constant) we get de Sitter space.

From the results of [16, 11] we know that the equations (2.2), (2.3) admit BPS black hole solutions (with naked singularities for negative cosmological constant) asymptotic to (2.4), and breaking half of the supersymmetry. These solutions are given by

$$ds^2 = -H^{-2}(1 + g^2 r^2 H^3)dt^2 + \frac{H}{1 + g^2 r^2 H^3}dr^2 + r^2 H d\Omega^2, \tag{2.5}$$

$$A_t = 3H^{-1}, \tag{2.6}$$

where $H = (1 + \frac{q}{r^2})$, and q is essentially the electric charge.

For $g = 0$, the metric (2.5) reduces to the extreme Reissner-Nordström black hole of five-dimensional Einstein-Maxwell gravity without cosmological constant, given by

$$ds^2 = -H^{-2}dt^2 + Hd\vec{x}^2, \tag{2.7}$$

where $d\vec{x}^2$ is the flat line element in four dimensions. (2.7) is of course the simplest example of the generalizations of Majumdar-Papapetrou solutions [17] to five dimensions, given by

$$\begin{aligned} ds^2 &= -\Omega^{-2}dt^2 + \Omega d\vec{x}^2, \\ A_t &= 3\Omega^{-1}, \quad \Omega = 1 + \sum_{j=1}^{N}\frac{q_j}{|\vec{x} - \vec{x}_j|^2}. \end{aligned}$$

By introducing the coordinate $R^2 = r^2 + q$, the solution (2.5) can be written in the form

$$ds^2 = -g^2 R^2 dt^2 - \frac{1}{R^4}(R^2 - q)^2 dt^2 + \frac{dR^2}{U(R)} + R^2 d\Omega^2, \tag{2.8}$$

where

$$U(R) = \left(1 - \frac{q}{R^2}\right)^2 + g^2 R^2. \tag{2.9}$$

We now present our solution which is the rotating generalization of (2.8). The equations of motion (2.2) and (2.3) are satisfied for the metric

$$ds^2 = -g^2 R^2 dt^2 - \frac{1}{R^4}((R^2 - q)dt - \alpha \sin^2\theta d\phi + \alpha \cos^2\theta d\psi)^2 + \frac{dR^2}{V(R)} + R^2 d\Omega^2, \tag{2.10}$$

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where

$$V(R) = \left(1 - \frac{q}{R^2}\right)^2 + g^2 R^2 - \frac{g^2 \alpha^2}{R^4}, \tag{2.11}$$

and the gauge fields

$$A_\phi = -3 \frac{\alpha \sin^2 \theta}{R^2}, \quad A_\psi = 3 \frac{\alpha \cos^2 \theta}{R^2}, \quad A_t = 3 \left(1 - \frac{q}{R^2}\right). \tag{2.12}$$

Notice in the above solution that g can be real and pure imaginary, corresponding to anti-de Sitter or de Sitter solutions.

3 Physical properties

In the following we will concentrate on de Sitter charged rotating black holes and their physical properties, and leave the study of anti-de Sitter solutions for a future publication.

3.1 Horizons

Horizons occur whenever $V(R) = 0$. This implies that[5]

$$\left(1 - \frac{q}{R^2}\right)^2 - g^2 R^2 + \frac{g^2 \alpha^2}{R^4} = 0. \tag{3.1}$$

This equation admits a solution representing a cosmological horizon for all parameter values. To study the issue of the existence of black hole event horizons, we first define the dimensionless parameters

$$a \equiv \alpha g^3, \quad \varrho \equiv q g^2. \tag{3.2}$$

One finds then that for $0 < 6\varrho < 1$ and

$$a_-^2(\varrho) < a^2 < a_+^2(\varrho), \tag{3.3}$$

where

$$a_\pm^2(\varrho) = -(\varrho^2 - \frac{2}{3}\varrho + \frac{2}{27}) \pm \frac{2}{27}(1 - 6\varrho)^{3/2}, \tag{3.4}$$

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[5]As we are interested in the de Sitter case, we substitute $g \to ig$ in (2.11).

one has a black hole with (inner) Cauchy horizon R_- and (outer) event horizon R_+. Clearly, there is also a cosmological horizon at $R = R_c > R_+$. For $a^2 = a^2_-(\varrho)$, the event and cosmological horizons coalesce, the resulting metric is then similar to the Nariai solution [18]. For $a^2 = a^2_+(\varrho)$, the Cauchy and event horizons coalesce, so that one has an extremal black hole in de Sitter space. Another interesting case appears for $6\varrho = 6\sqrt{3}a = 1$. The function $V(R)$ then reads

$$V(R) = g^2 R^2 \left(\frac{1}{3g^2 R^2} - 1 \right)^3,\tag{3.5}$$

which implies that we have in this case an "ultracold" cosmological horizon.

3.2   Temperature and entropy

We now wish to compute the Hawking and cosmological temperatures. To this end, we write the metric (2.10) in the canonical (ADM) form

$$ds^2 = -N^2 dt^2 + \sigma_{mn}(dx^m + N^m dt)(dx^n + N^n dt),\tag{3.6}$$

with the lapse function

$$N^2 = \frac{V(R)}{1 - \frac{\alpha^2}{R^6}},\tag{3.7}$$

and the shift vector

$$N^\phi = -N^\psi = \frac{\alpha(R^2 - q)}{R^6 - \alpha^2}.\tag{3.8}$$

Consider now the analytical continuation $t \to -i\tau$, which yields the "quasi-Euclidean" section

$$ds^2 = N^2 d\tau^2 + \sigma_{mn}(dx^m - iN^m d\tau)(dx^n - iN^n d\tau).\tag{3.9}$$

We assume to have either an event or a cosmological horizon at $R = R_H$, and use the expansion

$$V(R) = V'(R_H)(R - R_H)\tag{3.10}$$

near $R = R_H$. Defining the new coordinate

$$\tilde{R} = 2\sqrt{\frac{R - R_H}{V'(R_H)}}, \tag{3.11}$$

(3.9) can be written near $R = R_H$ as

$$ds^2 = d\tilde{R}^2 + \left(\frac{V'(R_H)}{2\sqrt{1 - \frac{\alpha^2}{R_H^6}}}\right)^2 \tilde{R}^2 d\tau^2 + R_H^2 d\theta^2 + \sigma_{ij}(dx^i - iN^i d\tau)(dx^j - iN^j d\tau), \tag{3.12}$$

where $i, j = \phi, \psi$. From (3.12), we see that the period of τ must be

$$\beta = \frac{1}{T} = \frac{4\pi}{|V'(R_H)|}\sqrt{1 - \frac{\alpha^2}{R_H^6}} \tag{3.13}$$

in order to avoid conical singularities. The angular velocities of the horizon read

$$\Omega_H^\phi = -\Omega_H^\psi = -N_H^\phi. \tag{3.14}$$

Note that in the case of zero cosmological constant, the angular velocities of the event horizon vanish [5], whereas for $g \neq 0$ the horizon rotates.

The Bekenstein-Hawking entropy of the de Sitter black holes under consideration is given by

$$S_{BH} = \frac{A_H}{4G_5} = \frac{\pi^2}{2G_5}(R_+^6 - \alpha^2)^{1/2}, \tag{3.15}$$

which reduces in the case $g = 0$ to the result found in [9].

3.3 Smarr formula

To obtain a Smarr-type formula (from which a first law of black hole mechanics can be deduced), we proceed along the lines of [19], where an analogous calculation for the four-dimensional Kerr-Newman-de Sitter black hole was performed. We start from the Killing identity

$$\nabla_\mu \nabla_\nu K^\mu = R_{\nu\rho} K^\rho = (F_{\nu\rho}^2 - \frac{1}{6}g_{\nu\rho}F^2)K^\rho + 4g^2 K_\nu, \tag{3.16}$$

where K^μ is a Killing vector, $\nabla_{(\nu}K_{\mu)} = 0$. Note that in the second step we used the Einstein equation of motion (2.2). Now integrate (3.16) on a spacelike hypersurface Σ_t

from the black hole horizon R_+ to the cosmological horizon R_c. On using Gauss' law this gives

$$\int_{\partial\Sigma_t} \nabla_\nu K_\mu d\Sigma^{\mu\nu} = 4g^2 \int_{\Sigma_t} K_\nu d\Sigma^\nu + \int_{\Sigma_t} (F_{\nu\rho}^2 - \frac{1}{6}g_{\nu\rho}F^2)K^\rho d\Sigma^\nu, \tag{3.17}$$

where the boundary $\partial\Sigma_t$ consists of the intersection of Σ_t with the black-hole and the cosmological horizon,

$$\partial\Sigma_t = S^3(R_+) \cup S^3(R_c). \tag{3.18}$$

In a first step, we apply (3.17) to the Killing vectors ∂_ϕ and ∂_ψ, which we denote by $\tilde{K}^i$, $i = \phi, \psi$. Using $\tilde{K}^i_\nu d\Sigma^\nu = 0$, we get

$$\frac{1}{8\pi} \int_{S^3(R_+)} \nabla_\nu \tilde{K}^i_\mu d\Sigma^{\mu\nu} + \frac{1}{8\pi} \int_{S^3(R_c)} \nabla_\nu \tilde{K}^i_\mu d\Sigma^{\mu\nu} = \int_{\Sigma_t} T_{\nu\rho} \tilde{K}^{i\rho} d\Sigma^\nu, \tag{3.19}$$

where

$$T_{\nu\rho} = \frac{1}{8\pi} \left(F_{\nu\rho}^2 - \frac{1}{4}g_{\nu\rho}F^2 \right) \tag{3.20}$$

is the stress-energy tensor of the vector field. One can interpret the right-hand side of (3.19) as the angular momentum of the matter between the two horizons. Thus one can regard the second term on the left-hand side of (3.19) as being the total angular momentum, J_c^i, contained in the cosmological horizon, and the first term on the left-hand side as the negative of the angular momentum J_{BH}^i of the black hole (cf. also discussion in [19]).

In a second step, we apply (3.17) to the Killing vector $K = \partial_t$. This yields

$$\frac{1}{4\pi} \int_{S^3(R_+)} \nabla_\nu K_\mu d\Sigma^{\mu\nu} + \frac{1}{4\pi} \int_{S^3(R_c)} \nabla_\nu K_\mu d\Sigma^{\mu\nu} =$$
$$\frac{g^2}{\pi} \int_{\Sigma_t} K_\nu d\Sigma^\nu + 2 \int_{\Sigma_t} \left(T_{\nu\rho} - \frac{1}{3}T g_{\nu\rho} \right) K^\rho d\Sigma^\nu. \tag{3.21}$$

One can regard the terms on the right-hand side of Eq. (3.21) as representing respectively the contribution of the cosmological constant and the contribution of the matter kinetic energy to the mass within the cosmological horizon. We therefore identify the second term on the left-hand side as the total mass M_c within the cosmological horizon, and the first term as the negative of the black hole mass M_{BH}. As in Ref. [20], the latter can be rewritten by expressing $K = \partial_t$ in terms of the null generator

$$l = \partial_t + \Omega_H^\phi \partial_\phi + \Omega_H^\psi \partial_\psi = \partial_t + \Omega_H^i \tilde{K}^i \tag{3.22}$$

of the black hole event horizon. Using the definition of the surface gravity κ, which, by the zeroth law, is constant on the horizon, one obtains

$$M_{BH} = \frac{\kappa A_H}{4\pi} + 2\Omega_H^i J_{BH}^i. \tag{3.23}$$

One therefore gets the Smarr-type formula

$$
\begin{aligned}
M_c &= \frac{\kappa A_H}{4\pi} + 2\Omega_H^\phi J_{BH}^\phi + 2\Omega_H^\psi J_{BH}^\psi + \frac{g^2}{\pi} \int_{\Sigma_t} K_\nu d\Sigma^\nu \\
&\quad + 2 \int_{\Sigma_t} \left(T_{\nu\rho} - \frac{1}{3} T g_{\nu\rho} \right) K^\rho d\Sigma^\nu.
\end{aligned} \tag{3.24}
$$

4   Cosmological multi-centered solutions

Multi-centered solutions of five-dimensional gravity with a cosmological constant have been discussed in [16] and more recently in [21] for the cases with vector multiplets. Multi-centered charged solutions in four dimensional de Sitter space were considered in [22].

In this section we generalize these solutions and find multi-centered rotating charged five-dimensional de Sitter black holes. In order to find these solutions, we recast the 5d de Sitter metric in the so-called cosmological coordinates, in which the metric appears similar to Minkowski space but with the Euclidean part multiplied by an exponential depending on time and the cosmological constant. This metric is given by

$$ds^2 = -dt^2 + e^{2gt}(dr^2 + r^2 d\Omega^2). \tag{4.1}$$

The metric of dS_5 given in (2.4) can be obtained from (4.1) by performing the change of variables

$$r' = re^{gt}, \quad dt = dt' - \frac{gr'}{1 - g^2 r'^2} dr'.$$

For our rotating charged black hole solution (2.10) we find that it can be written (after substituting $g \to ig$) in the form

$$ds^2 = -H^{-2}(dt - e^{-2gt}\frac{\alpha}{r^2}\sin^2\theta d\phi + e^{-2gt}\frac{\alpha}{r^2}\cos^2\theta d\psi)^2 + He^{2gt}(dr^2 + r^2 d\Omega^2), \tag{4.2}$$

$$A_\phi = -\frac{3\alpha \sin^2\theta}{r^2 H}e^{-2gt}, \quad A_\psi = \frac{3\alpha \cos^2\theta}{r^2 H}e^{-2gt}, \quad A_t = 3H^{-1},$$

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where we defined

$$H \equiv 1 + \frac{q}{r^2}e^{-2gt}.$$

(4.2) is related to the metric of (2.10) by a change of variables according to

$$
\begin{aligned}
r' &= re^{gt}, \quad dt = dt' + f(r')dr', \\
d\phi &= d\phi' + h(r')dr', \quad d\psi = d\psi' - h(r')dr',
\end{aligned}
$$

where

$$
\begin{aligned}
f(r') &= \frac{-gr'\left((1+\frac{q}{r'^2})^3 - \frac{\alpha^2}{r'^6}\right)}{1 - g^2 r'^2 (1+\frac{q}{r'^2})^3 + \frac{g^2\alpha^2}{r'^4}}, \\
h(r') &= \frac{\alpha g}{r'^3\left(1 - g^2 r'^2 (1+\frac{q}{r'^2})^3 + \frac{g^2\alpha^2}{r'^4}\right)},
\end{aligned}
$$

and subsequently setting $R^2 = r'^2 + q$ and dropping the primes. As for the unrotating solutions [16, 21], the harmonic function can be chosen to be

$$H = 1 + \sum_{j=1}^{N} \frac{q_j}{|\vec{x} - \vec{x}_j|^2}e^{-2gt}. \tag{4.3}$$

In this case one obtains a multi-centered solution representing an arbitrary number of charged rotating black holes in a space-time with positive cosmological constant.

5 Final remarks

In this work we have obtained new solutions of Einstein-Maxwell gravity with Chern-Simons term and cosmological constant. Some physical properties of the rotating charged de Sitter solutions have also been studied. We also found the coordinate transformation which recast our de Sitter solution in the so-called cosmological coordinates. In the cosmological coordinates one finds the generalization of our solution to the multi-centered case.

As in the unrotating case, we note here that our rotating anti-de Sitter solution is BPS and breaks half of the supersymmetry of the $N = 2$, $d = 5$ gauged supergravity theory. This means that our solution solves Killing spinor equation,

$$\left(\mathcal{D}_\mu + \frac{i}{24}(\Gamma_\mu{}^{\nu\rho} - 4\delta_\mu{}^\nu\Gamma^\rho)F_{\nu\rho} + \frac{1}{2}g\Gamma_\mu - \frac{i}{2}gA_\mu\right)\epsilon = 0. \tag{5.1}$$

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Here $\mathcal{D}_\mu$ is the gauge and gravitationally covariant derivative. The Killing spinor is the same as in the unrotated case [16, 21],

$$\epsilon = H^{-\frac{1}{2}} e^{igt} \epsilon_0, \tag{5.2}$$

where H is given by (4.3), and ϵ_0 is a constant spinor satisfying the constraint $\Gamma_0 \epsilon_0 = i\epsilon_0$. Clearly, the de Sitter solution admits a Killing spinor which can be obtained by replacing g with ig in the above expressions.

We should also note here that the rotating solution in anti-de Sitter space has a horizon unlike the static solutions which have a naked singularity. The supersymmetry properties of the charged rotating AdS black holes as well as their relevance to the AdS/CFT correspondence are currently under study.

Acknowledgements

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